5th Week Friday

24 Mart 2023 Cuma 10:26

Adjoint Matrix $adj(A)$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \quad \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}_{n\times n} \xrightarrow{\text{take transpose !}} \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}_{n\times n} = adj(A)$$

i,j^{th} cofactor $\rightarrow A_{ij} \rightarrow \underbrace{(-1)^{i+j}}_{-,+} M_{ij} \rightarrow | \quad |$ delete i^{th} row, j^{th} column from A

Ex

$$A = \begin{bmatrix} 1 & 5 & 3 \\ -2 & -4 & -3 \\ -3 & -5 & 1 \end{bmatrix}_{3\times 3} \quad adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -19 & -20 & -3 \\ 11 & 10 & -3 \\ -2 & -10 & 6 \end{bmatrix} \checkmark$$

$A_{11} = \underbrace{(-1)^{1+1}}_{+} \cdot M_{11} \quad \begin{vmatrix} -4 & -3 \\ -5 & 1 \end{vmatrix} = -4-(15) = -19$

$A_{12} = \underbrace{(-1)^{1+2}}_{-} M_{12} \quad \begin{vmatrix} -2 & -3 \\ -3 & 1 \end{vmatrix} = -2-9 = -11$

$A_{13} = \underbrace{(-1)^{1+3}}_{+} M_{13} \quad \begin{vmatrix} -2 & -4 \\ -3 & -5 \end{vmatrix} = 10-12 = -2$

$A_{21} = \underbrace{(-1)^{2+1}}_{-} \cdot M_{21} \quad \begin{vmatrix} 5 & 3 \\ -5 & 1 \end{vmatrix} = 5-(-15) = 20$

$A_{22} = \underbrace{(-1)^{2+2}}_{+} M_{22} \quad \begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} = 1-(-9) = 10$

$A_{23} = \underbrace{(-1)^{2+3}}_{-} M_{23} \quad \begin{vmatrix} 1 & 5 \\ -3 & -5 \end{vmatrix} = -5-(-15) = 10$

$A_{31} = \underbrace{(-1)^{3+1}}_{+} \cdot M_{31} \quad \begin{vmatrix} 5 & 3 \\ -4 & -3 \end{vmatrix} = -15-(-12) = -3$

$A_{32} = \underbrace{(-1)^{3+2}}_{-} M_{32} \quad \begin{vmatrix} 1 & 3 \\ -2 & -3 \end{vmatrix} = -3-(-6) = 3$

$A_{33} = \underbrace{(-1)^{3+3}}_{+} M_{33} \quad \begin{vmatrix} 1 & 5 \\ -2 & -4 \end{vmatrix} = -4-(-10) = 6$

Application of the Adjoint Matrix on finding A^{-1}

$$A \cdot adj(A) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}_n = \begin{bmatrix} det(A) & 0 & 0 \cdots & 0 \\ 0 & det(A) & & 0 \\ & 0 & \ddots & \\ & & & det(A) \end{bmatrix} = \underbrace{det(A)}_{\in \mathbb{R}} \cdot \underbrace{I_n}_{\text{scal}}$$

$\checkmark \; a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n} = det(A)$

wrong cof. exp $\rightarrow a_{11}A_{21} + a_{12}A_{22} + \cdots + a_{1n}A_{2n} = 0$

$$A \cdot adj(A) = det(A) \, I_n$$

$$A \cdot \operatorname{adj}(A) = \det(A) \cdot I_n$$

$$A \underbrace{\frac{1}{\det(A)} \operatorname{adj}(A)}_{A^{-1}} = I_n$$

✓✓

Ex

$\det(A) =$ ....